Determination of Digitizer Absolute Phase Using Equivalent Time Sampling

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***Abstract*—The rigorous characterization of the absolute phase in digitization systems still represents an open issue for national metrological institutes, research laboratories and manufacturers. In this paper, we present a new method for the determination of absolute phase, based on curve fitting a model of a filtered square wave, in order to determine its phase related to the phase of a sampled sine wave at the same frequency. The square wave is generated by a digital clocking system with a fast rise time. The square wave is input to the digitizer being calibrated. The anti-aliasing filter of the digitizer rounds off the rising and falling edges based on the filter's frequency response. The equivalent time sampling (ETS) technique is used to capture the shape of the filtered rising and falling edges of the signal, and the ETS captured signal is modeled.**

I. Introduction

The term of art *"absolute phase"* has been in use since the mid-1800's to mean the phase shift through a process [1], [2]. In 2002, it was used in reference to the phase of an electric power signal relative to the pulse per second (PPS) output from global navigation satellite systems (GNSS) [3]. Since then, *absolute phase* in bulk electrical power systems has increasingly referred to the power system phase angle relative to a theoretical cosine wave at the power system nominal frequency and coincident with the coordinated universal time (UTC) second [4]. This paper uses the term in both the information theory sense to mean the phase angle difference between the input and output signals of a process, and in the power systems' sense in that the input signal is synchronous in time and coincident with the international atomic time (UTC/TAI) second realized by the PPS.

An absolute phase calibration system is shown in Fig. 1. In this system, the original signal has been replaced by a function generator (FGEN). The FGEN's digital-to-analog converter (DAC) obtains its time-base and trigger from the same clock that is also driving the ADC of the digitizer being tested. Additionally, the time reference provides a trigger whose triggering edge coincides with the UTC/TAI second. The time reference also provides a square wave (whose rise time is negligible compared to the time constant of the digitizer's anti-aliasing filter)[1]. This square wave will be called the phase reference signal (PRS) [5]. The FGEN provides a sinusoid at the same frequency as the PRS. The frequencies generated are integer cycles per second. By assessing the delay between the UTC second and the sampled PRS signals, it is possible to determine the absolute phase shift of the digitization system.

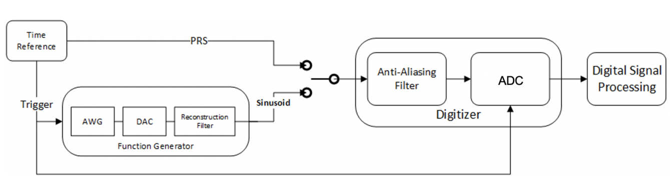

Fig. 1. Block scheme of an absolute phase calibration system.

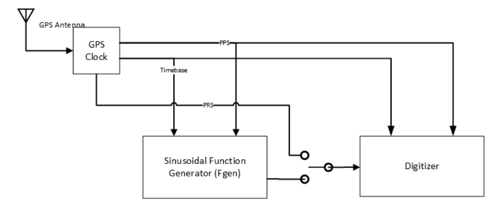

Fig. 2. Block scheme of NIST absolute phase determination system.

This paper proposes to use the equivalent time sampling (ETS) technique to capture very high time-resolution sample sets for the sampled PRS captured by the digitizer, and use the ETS sample set to determine a model of the PRS convolved with the digitizer's anti-aliasing filter.

II. Test Setup

As shown in Fig. 2, the test system has a GPS clock and time reference system which provides the sampler timebase and the pulse-per-second (PPS) trigger to both the digitizer and the function generator. The GPS clock also provides a GPS-synchronized, phase reference signal that can be programmed in frequency and phase with a start time relative to the PPS.

The PPS is used as a trigger and the ultimate time reference. The PPS alignment to UTC is guaranteed up to ± 30 ns.

III. Proposed Method

NIST, looking to improve its own 200 ns delay uncertainty, began by evaluating methods proposed by METAS-EPFL [6], VSL [7], and INRiM [5]. A concern with the METAS-EPFL method was that with a 100 kHz digitizer sampling rate, the determination of the actual phase using IpDFT would not yield the resolution desired for such uncertainty. The VSL method uses pure FFT and looks at the corresponding bins requiring integer numbers of cycles to avoid leakage and very large sample sets to reduce bin size. The VSL method resolution also depends on the sampling rate. The INRiM method looked very promising with its use of the counter/phase comparator. This method depends on the ADC timing signals (trigger and sample) clock to always be available, which may not

[1] In the case of our test system, the rise time is from 0.5 ns to 2.5 ns.

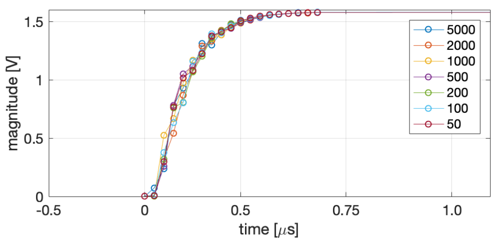

Fig. 3. ETS acquisition of PRS waveforms with different PRS frequencies.

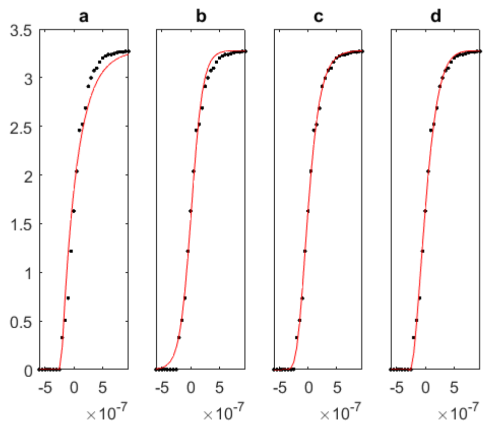

Fig. 4. a) exponential model, b) logistic model, c) Richards growth curve, d) Weibull CDF model

be the case for all systems. Furthermore, the *sampled* PRS is assumed to be the reference signal; however, it has been further delayed by the anti-aliasing filter before sampling. The method proposed herein is a refinement of the INRiM method, accounting for the group delay and roll-off of the filter by using the ETS technique to determine a model of the PRS square wave convolved with the anti-aliasing filter, and then fitting that model to the sampled PRS to determine the digitizer's delay. This method does not require the ADC internal timing signals to be available.

a) Defining a model of the filtered PRS: We analyze the digitizer anti-aliasing filter's phase shift and roll-off of the square edges of the PRS. The ETS method is used to determine this response even though the time constant of the filter is much faster than the sampling rate of the digitizer.

We evaluated whether the sampled PRS rising edge depends on its frequency, f_{PRS}. Fig. 3 presents the PRS waveforms acquired at different frequencies and shows that the anti-aliasing filter group delay is not frequency dependent within the observed bandwidth of 50 Hz to 5000 Hz.

b) Model determination: We evaluated several models to find one that not only provides a good fit of the data but could also be used to mathematically determine the delay. We looked at exponential, logistic, Richard's curve [8], Birch [9], and Weibull [10] functions.

Fig. 4 shows the evaluated models fitted using the Nelder-Mead technique [11], Upon close inspection of the exponential fit, it became apparent that the shape of the ETS sampled data was more of a sigmiod. The logistic curve, with its inflection point in the center was attempted. Upon inspection of that, the inflection point was seen to be lower than center so Richards curve was fitted. The inflection point is quite low however and the important part of the fit, at the beginning of the curve, did not fit well. This issue was addressed by Birch in his 1999 paper, but we had a difficult time fitting his curve since the analytic function cannot be empirically derived. Birch led us to the Weibull function which not only provides an acceptable fit, but the time offset parameter is exactly the delay figure so no further derivation or calculation is needed.

The Weibull function is given by:

$$f(t) = a \cdot [1 - e^{(-\frac{t-t_0}{\lambda})^{\nu}}] \qquad (1)$$

for the rising edge and:

$$f(t) = a \cdot [e^{(-\frac{t-t_0}{\lambda})^{\nu}}] \qquad (2)$$

for the falling esge where the parameters a is the signal amplitude, ν is the shape parameter determining the placement of the inflection point and λ is a scale parameter. t_0 is the beginning of the growth so the value of this parameter is the delay result we need with no further calculations needed.

IV. Conclusions

In this paper, we present a method for the assessment of the absolute phase of high-accuracy digitization systems. The proposed method adopts a square-wave Phase Reference Signal (PRS), locked to the UTC/TAI second, to distribute a programmable and traceable time reference within the different stages of the system under test. In order to improve the estimation resolution and accuracy, the Equivalent Time Sampling (ETS) technique is used to acquire the PRS at each of the frequencies of interest. This yields a high resolution model of the sampled PRS convolved with the digitizer, including the digitizer's anti-aliasing filter.

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